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Design of Stretchable Electronics Against Impact

Stretchable electronics offer soft, biocompatible mechanical properties; these same properties make them susceptible to device failure associated with physical impact. This paper studies designs for stretchable electronics that resist failure from impacts due to incorporation of a viscoelastic encapsulation layer. Results indicate that the impact resistance depends on the thickness and viscoelastic properties of the encapsulation layer, as well as the duration of impact. An analytic model for the critical thickness of the encapsulation layer is established. It is shown that a commercially available, low modulus silicone material offers viscous properties that make it a good candidate as the encapsulation layer for stretchable electronics. [DOI: 10.1115/1.4034226]

Keywords: stretchable electronics, impact loading, viscoelasticity

Introduction

Stretchable electronics allow conformal contact with nondevelopable surfaces of the human skin or internal tissues/organs without degradation of device performance [1–10]. These systems are designed to accommodate large stretching, bending, and twisting motions along the skin surface, but with no special considerations for those normal to the surface. In practical applications, several underlying mechanisms may reduce the reliability of flexible/stretchable electronics, such as the mechanical fatigue due to long-term periodic activities of human muscles [11,12], or the electromigration in metallic interconnects at relatively high current densities [13,14]. In addition, it is inevitable that stretchable electronics will suffer from unexpected collisions or impacts when worn on the skin [15–17], leading to possible device failure such as the rupture of active silicon materials at high stress levels, the irreversible deformation of metallic interconnects under complex loading conditions, or the interfacial slippage and delamination due to stiffness mismatch [18–22]. Designs that minimize damage in such contexts are, therefore, important to

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Contributed by the Applied Mechanics Division of ASME for publication in the Journal of Applied Mechanics. Manuscript received June 29, 2016; final manuscript received July 15, 2016; published online August 10, 2016. Assoc. Editor: Pradeep Sharma.

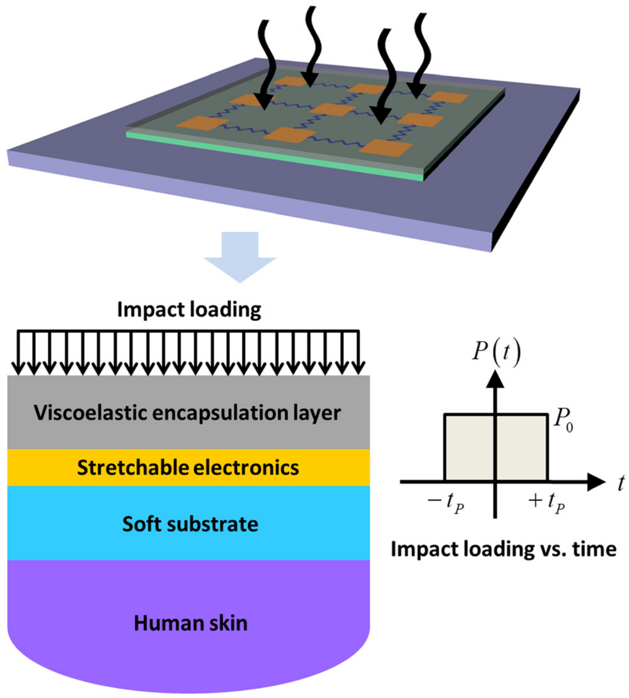

Fig. 1 One-dimensional model of the stretchable electronics, with viscoelastic encapsulation and soft substrate, on the surface of human skin subjected to impact loading

consider. This paper studies designs that incorporate a viscoelastic encapsulation layer on top of stretchable electronics to bear the impact loading and to dissipate the impact energy through viscoelastic effects. Guidelines in terms of the thickness and modulus of the viscoelastic encapsulation layer are provided to reduce the magnitude of the impact transferred to the devices.

Model Description and Analysis

The system examined here involves an applied normal impact load that can be considered uniform across the device, which suggests a simple, 1D model as shown in Fig. 1. A device that incorporates fragile components (e.g., made of silicon) on a soft substrate, encapsulated by a viscoelastic layer, rests on the surface of the skin, with the soft substrate in direct contact with the surface of the skin. A uniformly distributed impact loading is applied to the surface of the encapsulation layer. Let P_0 and $2t_P$ denote the magnitude and time duration of the rectangular-shaped impact stress, respectively (Fig. 1). This paper determines the critical thickness h or modulus E of the encapsulation layer to attenuate the impact from P_0 to εP_0 due to viscoelasticity when the impact reaches the fragile components, where $\varepsilon \ll 1$.

Viscoelastic Constitutive Model. The viscoelastic properties of the soft encapsulation layer can be characterized by the frequency-dependent complex modulus E as

$$E(\omega) = E_{\text{storage}}(\omega) + \mathrm{i}E_{\text{loss}}(\omega) = |E(\omega)|e^{\mathrm{i}\delta(\omega)} \tag{1}$$

where E_{storage} and E_{loss} are the storage and loss moduli, respectively; $|E|$ is the magnitude of E; δ is the loss angle; "i" is the imaginary unit; and ω is the angular frequency of the harmonic vibration. The storage modulus and magnitude are extended as even functions of ω, $E_{\text{storage}}(-\omega) = E_{\text{storage}}(\omega)$ and $|E(-\omega)| = |E(\omega)|$, whereas the loss modulus and angle are odd functions, $E_{\text{loss}}(-\omega) = -E_{\text{loss}}(\omega)$ and $\delta(-\omega) = -\delta(\omega)$.

For a polymer encapsulation layer, its complex modulus can be written empirically in the Havriliak–Negami type as [23,24]

$$E = E_i - \frac{E_i - E_r}{1 + (i\omega t_r)^{\alpha}}, \quad \omega \geq 0, \quad E_i > E_r, \quad 0 < \alpha < 1 \tag{2}$$

where E_i and E_r are the instantaneous and relaxed moduli, respectively, t_r is the relaxation time, governing how fast the time-varying relaxation modulus transits from E_i to E_r in the viscoelastic stress-relaxation process, and the limits $\alpha = 0$ and $E_r = E_i$ all correspond to vanishing viscoelasticity. The real and imaginary parts of Eq. (2) give the storage and loss moduli in Eq. (1), respectively, as

$$E_{\text{storage}} = \frac{E_r + (E_i + E_r)(\omega t_r)^{\alpha}\cos\dfrac{\alpha\pi}{2} + E_i(\omega t_r)^{2\alpha}}{1 + 2(\omega t_r)^{\alpha}\cos\dfrac{\alpha\pi}{2} + (\omega t_r)^{2\alpha}} \leq E_i$$

$$E_{\text{loss}} = \frac{(E_i - E_r)(\omega t_r)^{\alpha}\sin\dfrac{\alpha\pi}{2}}{1 + 2(\omega t_r)^{\alpha}\cos\dfrac{\alpha\pi}{2} + (\omega t_r)^{2\alpha}} \leq \frac{E_i - E_r}{2}\tan\frac{\alpha\pi}{4}, \quad (\omega \geq 0) \tag{3}$$

in which E_{loss} reaches its maximum when $\omega t_r = 1$. For $\omega \to 0$ or $\omega \to \infty$, the loss modulus $E_{\text{loss}} \sim 0$ and the storage modulus E_{storage} approaches E_r and E_i, respectively.

In view of Eq. (3), the loss tangent $E_{\text{loss}}/E_{\text{storage}}$ satisfies the following inequality:

$$\frac{(E_i - E_r)\sin\dfrac{\alpha\pi}{2}}{2\sqrt{E_iE_r} + (E_i + E_r)\cos\dfrac{\alpha\pi}{2}} \geq \frac{E_{\text{loss}}}{E_{\text{storage}}} \geq \begin{cases} \eta(\omega t_r)^{-\alpha}, & \text{if } \omega t_r \geq 1 \\ \eta(\omega t_r)^{\alpha}, & \text{if } 0 \leq \omega t_r \leq 1 \end{cases} \tag{4}$$

with $\eta = (E_i + E_r)^{-1}(E_i - E_r)\tan(\alpha\pi/4)$, and the loss tangent $E_{\text{loss}}/E_{\text{storage}}$ reaches the maximum when $(\omega t_r)^{\alpha} = \sqrt{E_r/E_i}$.

Viscoelastic Penetration Depth of 1D Harmonic Loadings. The viscoelastic penetration depth is first studied in the following manner. For a harmonic normal stress $\sigma_{\text{surface}} = \sigma_0 e^{\mathrm{i}\omega t}$, with magnitude σ_0, frequency ω, and time t, applied uniformly on the surface of a semi-infinite viscoelastic soft material, the normal stress σ and displacement u within the viscoelastic material can be expressed as $\sigma = \sigma_{\text{surface}}e^{-\mathrm{i}kx} = \sigma_0 e^{\mathrm{i}(\omega t - kx)}$ and $u = u_0 e^{\mathrm{i}(\omega t - kx)}$, where k and u_0 are complex numbers to be determined, and x (≥ 0) is the spatial coordinate (with $x = 0$ being the surface). Substitution of σ and u into the equilibrium equation and the constitutive model below

$$\frac{\partial \sigma}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2} \quad \text{and} \quad \sigma = E\frac{\partial u}{\partial x} \tag{5}$$

(with ρ being the mass density) gives

$$k = k(\omega) = \frac{\omega}{c(\omega)}e^{-\mathrm{i}\delta(\omega)/2}, \quad c(\omega) = \sqrt{|E(\omega)|/\rho} \tag{6}$$

The stress can then be obtained as

$$\sigma = \sigma_{\text{surface}}e^{-x/d}e^{-\mathrm{i}Kx}, \quad K = K(\omega) = -K(-\omega) = \frac{\omega}{c(\omega)}\cos\frac{\delta(\omega)}{2} \tag{7}$$

where the viscoelastic penetration depth is given by

$$d = d(\omega) = d(-\omega) = \frac{c(\omega)}{\omega\sin\dfrac{\delta(\omega)}{2}} \tag{8}$$

Equation (7) suggests that the exponential decay of the magnitude of the harmonic stress with frequency ω is characterized by the viscoelastic penetration depth d. For $\omega \to 0$ and the modulus in Eq. (2), the viscoelastic penetration depth d in Eq. (8) approaches infinity, which indicates that the time-harmonic surface loading with extremely low frequency penetrates into the viscoelastic material easily. The viscoelastic penetration depth d approaches zero for $\omega \to \infty$, which indicates that it is very difficult for the time-harmonic surface loading with extremely high frequency to penetrate into the viscoelastic material.

Normal Stress due to the Rectangular-Shaped Impact Loading. The rectangular-shaped impact loading (Fig. 1) can be written as the integral of harmonic components with frequency ω ranging from $-\infty$ to $+\infty$, i.e.,

$$\frac{P(t)}{P_0} = \begin{cases} 1, & -t_P < t < +t_P; \\ 0, & \text{otherwise}; \end{cases} = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\sin(\omega t_P)}{\omega} e^{\mathrm{i}\omega t} \mathrm{d}\omega \tag{9}$$

In view of Eq. (7), the induced impact stress in the viscoelastic soft material can then be obtained similarly as

$$\begin{aligned} \frac{P(x,t)}{P_0} &= \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\sin(\omega t_P)}{\omega} e^{-x/d} e^{-\mathrm{i}Kx} e^{\mathrm{i}\omega t} \mathrm{d}\omega \\ &= \frac{2}{\pi}\int_{0}^{+\infty} \frac{\sin(\omega t_P)}{\omega} e^{-x/d} \cos(\omega t - Kx) \mathrm{d}\omega \end{aligned} \tag{10}$$

where d and K are functions of ω given in Eqs. (8) and (7), respectively. The above integral can be bounded by a series of inequalities as demonstrated in the following. Equation (10) can be expressed via the inequality $|a+b| \le |a| + |b|$ as

$$\left|\frac{P(x,t)}{P_0}\right| \le \frac{2}{\pi}\int_0^{1/t_r} \frac{|\sin(\omega t_P)|}{\omega} e^{-x/d} \mathrm{d}\omega + \frac{2}{\pi}\int_{1/t_r}^{+\infty} \omega^{-1} e^{-x/d} \mathrm{d}\omega \tag{11}$$

The last integral in Eq. (11), with d given in Eq. (8), can be bounded using Eq. (4) and the inequality $\sin(\theta/2) \ge \theta\theta_0^{-1}\sin(\theta_0/2) \ge \tan\theta(\tan\theta_0)^{-1}\sin(\theta_0/2)$ for $0 \le \theta \le \theta_0 \le \pi/4$ with $\theta = \arctan[\eta(\omega t_r)^{-\alpha}]$ and $\theta_0 = \arctan\eta$ when $\omega t_r \ge 1$

$$\begin{aligned} \int_{1/t_r}^{+\infty} \omega^{-1} e^{-x/d} \mathrm{d}\omega &= \int_{1/t_r}^{+\infty} \omega^{-1} e^{\frac{-x\omega}{c(\omega)}\sin\left[\frac{1}{2}\arctan\frac{E_{\text{loss}}(\omega)}{E_{\text{storage}}(\omega)}\right]} \mathrm{d}\omega \\ &\le \int_{1/t_r}^{+\infty} \omega^{-1} e^{\frac{-x\omega}{c(\omega)}\sin\left\{\frac{1}{2}\arctan\left[\eta(\omega t_r)^{-\alpha}\right]\right\}} \mathrm{d}\omega \\ &\le \int_{1/t_r}^{+\infty} \omega^{-1} e^{\frac{-x\omega}{c(\omega)}\frac{\eta(\omega t_r)^{-\alpha}}{\sqrt{\left(1+\sqrt{1+\eta^2}\right)^2+\eta^2}}} \mathrm{d}\omega \end{aligned} \tag{12}$$

Substitution of Eq. (3) into Eq. (6) gives

$$\begin{aligned} c(\omega) &= \rho^{-1/2}\left(E_{\text{storage}}^2 + E_{\text{loss}}^2\right)^{1/4} \\ &\le \rho^{-1/2}\left[E_i^2 + \left(\frac{E_i - E_r}{2}\tan\frac{\alpha\pi}{4}\right)^2\right]^{1/4} \end{aligned} \tag{13}$$

Its substitution into the last integral in Eq. (12) gives

$$\begin{aligned} &\int_{1/t_r}^{+\infty} \omega^{-1} e^{\frac{-x\omega}{c(\omega)}\frac{\eta(\omega t_r)^{-\alpha}}{\sqrt{\left(1+\sqrt{1+\eta^2}\right)^2+\eta^2}}} \mathrm{d}\omega \le \int_{1/t_r}^{+\infty} \omega^{-1} e^{-(\omega t_r)^{1-\alpha}\chi} \mathrm{d}\omega \\ &= \frac{1}{1-\alpha}\int_{\chi}^{+\infty} z^{-1} e^{-z} \mathrm{d}z = -\frac{1}{1-\alpha}\mathrm{Ei}(-\chi) \end{aligned} \tag{14}$$

where $\mathrm{Ei}(z) = \int_{-\infty}^{z} z^{-1} e^{z} \mathrm{d}z$ is the so-called exponential integral in z [25], and χ is defined by

$$\begin{aligned} \chi &= \frac{\eta X}{\sqrt{\left(1+\sqrt{1+\eta^2}\right)^2+\eta^2}}\left[1 + \left(\frac{1 - E_r/E_i}{2}\tan\frac{\alpha\pi}{4}\right)^2\right]^{-1/4} \\ X &= \frac{x}{t_r\sqrt{E_i/\rho}}, \quad \eta = \frac{E_i - E_r}{E_i + E_r}\tan\frac{\alpha\pi}{4} \end{aligned} \tag{15}$$

Substitution of Eq. (14) into Eq. (12) gives

$$\int_{1/t_r}^{+\infty} \frac{1}{\omega} e^{-x/d} \mathrm{d}\omega \le -\frac{1}{1-\alpha}\mathrm{Ei}(-\chi) \tag{16}$$

With a change of variable $\Omega = \omega t_r$, the other integral in Eq. (11) can be rewritten via Eq. (15) as

$$\begin{aligned} &\int_0^{1/t_r} \frac{|\sin(\omega t_P)|}{\omega} e^{-x/d} \mathrm{d}\omega \equiv I \\ &= \int_0^1 |\sin(\Omega t_P/t_r)| \Omega^{-1} e^{-\Omega X \frac{E_{\text{loss}}\sqrt{E_i/2}}{\sqrt{E_{\text{storage}}^2 + E_{\text{loss}}^2}\sqrt{E_{\text{storage}} + \sqrt{E_{\text{storage}}^2 + E_{\text{loss}}^2}}}} \mathrm{d}\Omega \end{aligned} \tag{17}$$

in which E_{storage} and E_{loss} are functions of $\Omega = \omega t_r$ as given in Eq. (3), and X is the normalized position given in Eq. (15). The finite, nonsingular integral above, denoted by I, can be evaluated numerically, and it depends on the normalized position X, the dimensionless viscoelastic properties E_r/E_i and α, and the ratio of viscoelastic relaxation time to (half of the) duration of impact t_r/t_P. Substitution of Eqs. (16) and (17) into Eq. (11) finally gives

$$\left|\frac{P(x,t)}{P_0}\right| \le \frac{2}{\pi}\left[I - \frac{1}{1-\alpha}\mathrm{Ei}(-\chi)\right] \tag{18}$$

where I is given in Eq. (17). The above equation is used in the following to determine the critical thickness and modulus of the encapsulation layer for protection against normal impacts in the design of stretchable electronics.

Critical Thickness of the Single Encapsulation Layer. The right-hand side of Eq. (18) bounds the magnitude of the impact response, in which χ or X defined by Eq. (15) is the only parameter associated with the critical thickness of the single encapsulation layer (i.e., $x = h$). For each χ or X, $|P(x,t)/P_0|$ is bounded by Eq. (18). Setting its right-hand side

$$\frac{2}{\pi}\left[I - \frac{1}{1-\alpha}\mathrm{Ei}(-\chi)\right] = \varepsilon \tag{19}$$

ensures $\max\{|P(x,t)/P_0|\} < \varepsilon$. The above equation gives the critical thickness h of the encapsulation layer in terms of ε. The relation between ε and the normalized thickness X defined in Eq. (15) (with x replaced by h) also depends on three dimensionless viscoelastic properties, E_r/E_i, α, and t_r/t_P.

Figure 2 shows that the thickness of the encapsulation layer obtained from the simple estimate in Eq. (19) agrees well with the exact relation in Eq. (10) for $\alpha = 0.5$, $E_r/E_i = 0.1$ and two representative values of $t_r/t_P = 0.01$ and 0.1. Equation (19) clearly provides a simple but good estimate.

Figures 3–5 show ε versus the normalized thickness X in Eq. (15) (with x replaced by h) for varying E_r/E_i, α, and t_r/t_P, respectively. It is clear that the viscoelastic effect (decreasing E_r/E_i in Fig. 3 and increasing α in Fig. 4) reduces ε, and so does a short duration of impact (increasing t_r/t_P in Fig. 5), which all yield a smaller critical thickness h of the encapsulation layer for a

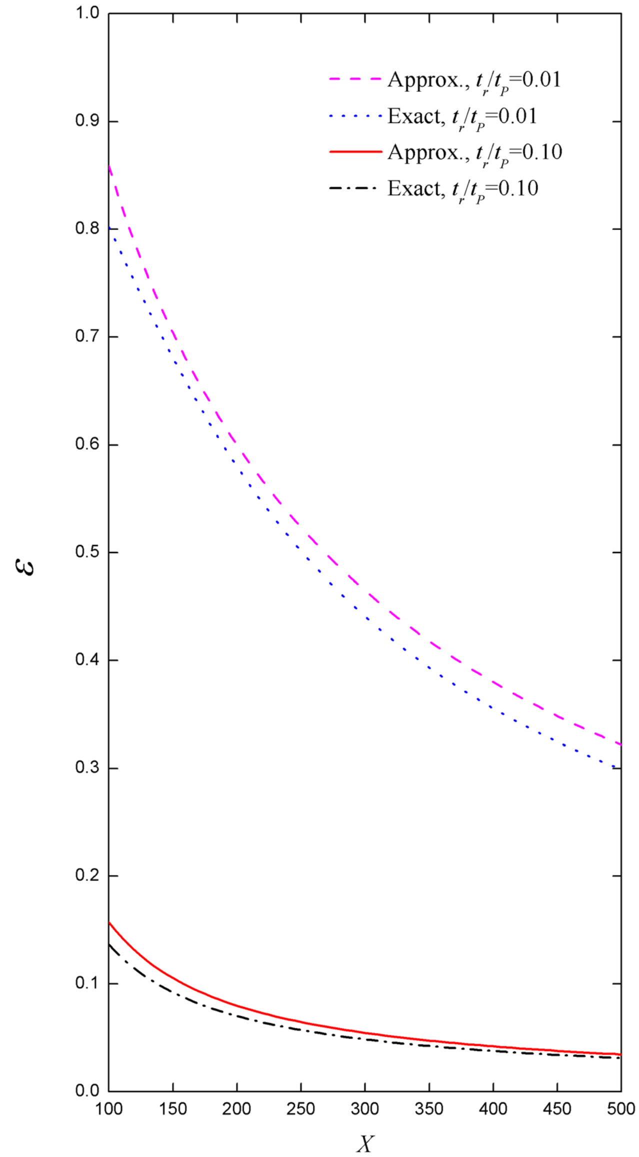

Fig. 2 ε **versus** X **for** $t_r/t_P = 0.01$ **and 0.1 with** $E_r/E_i = 0.1$ **and** $\alpha = 0.5$**, based on Eq. (19) ("Approx.") and Eq. (10) ("Exact"), respectively**

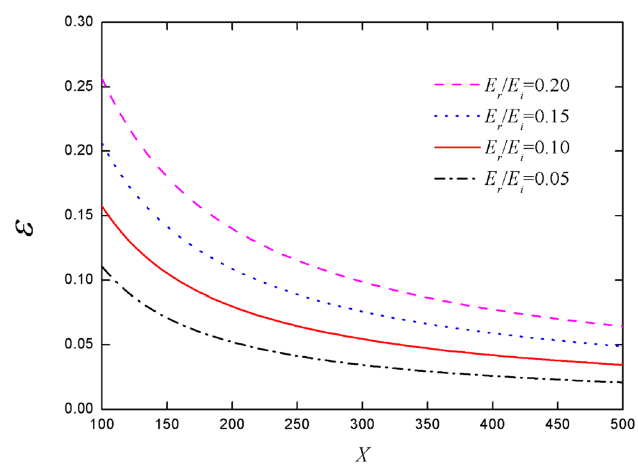

Fig. 3 ε **versus** X **for several values of** E_r/E_i **with** $\alpha = 0.5$ **and** $t_r/t_P = 0.1$

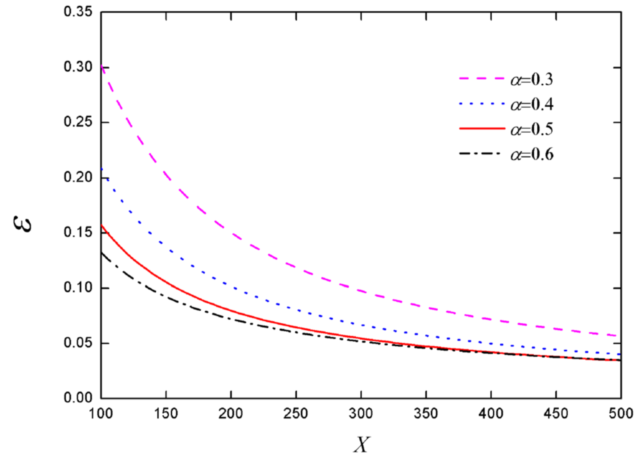

Fig. 4 ε **versus** X **for several values of** α **with** $E_r/E_i = 0.1$ **and** $t_r/t_P = 0.1$

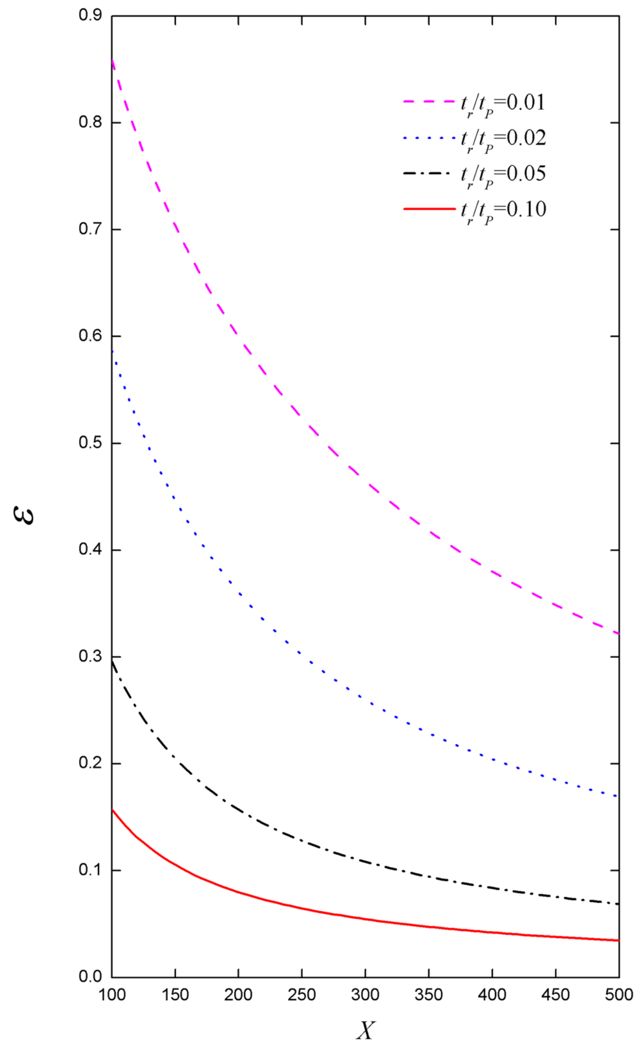

Fig. 5 ε **versus** X **for several values of** t_r/t_P **with** $E_r/E_i = 0.1$ **and** $\alpha = 0.5$

given ε. For the baseline values $E_r/E_i = 0.1$ and $\alpha = 0.5$, relaxation modulus $E_r = 3$ kPa and mass density $\rho \approx 1000\,\mathrm{kg/m^3}$ [26] of Silbione (a commercially available soft, viscous material based on a silicone chemistry), and the representative values of the relaxation time $t_r \sim 1\,\mu$s and duration of impact $t_P = 10\,\mu$s, Eq. (19) gives $X \approx 160$ for the reduction of impact by ten times (i.e., $\varepsilon = 0.1$), which leads to a thickness $\sim$0.9 mm, a practical value for the encapsulation layer in stretchable electronics. This result suggests that Silbione may be a good candidate as the encapsulation layer of the stretchable electronics against impact.

Conclusions

An analytic formula is derived in this paper to estimate the critical thickness of the viscoelastic encapsulation layer for protecting stretchable electronics against impact loadings. It is shown that the reduced impact to stretchable electronics depends on the thickness and viscoelastic properties of the encapsulation layer, as well as the duration of impact. Silbione, an extremely soft and viscous material, is suggested as a good candidate for the encapsulation layer of stretchable electronics.

Acknowledgment

J.Y. acknowledges the support from the National Natural Science Foundation of China (Grant No. 11402133). X.F. acknowledges the support from the National Basic Research Program of China (Grant No. 2015CB351900) and the National Natural Science Foundation of China (Grant No. 11320101001). Y.H. acknowledges the support from the U.S. National Science Foundation (DMR-1121262, CMMI-1300846, and CMMI-1400169) and from the U.S. National Institutes of Health (Grant No. R01EB019337).

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