A Mechanics Model for Sensors Imperfectly Bonded to the Skin for Determination of the Young's Moduli of Epidermis and Dermis

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A mechanics model is developed for the encapsulated piezoelectric thin-film actuators/sensors system imperfectly bonded to the human skin to simultaneously determine the Young's moduli of the epidermis and dermis as well as the thickness of epidermis.
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Introduction

The overall mechanical properties of the human skin depend mainly on the nature and organization of the dermal collagen and elastic fiber network, water, and proteins [1]. Studies of the mechanical properties of skin, such as Young's modulus, can provide an assessment in diagnosis and rehabilitation of dermal diseases. The complex stratified structure of the human skin adds many restrictions in the experiments for measuring the Young's modulus. Conventional methods including suction [1,2], indentation [3], traction [4], torsion [5], and wave propagation [6] provide useful insight into the averaged mechanical behavior of human skin but are problematic in terms of extracting the Young's moduli of the epidermis and dermis.

Stretchable and flexible electronics have been developed to measure the electrophysiological signals and mechanical properties of the human body [7–15]. Recently, Dagdeviren et al. [16] presented a microscale, conformal piezoelectric system for measuring the modulus of the human epidermis, which provides soft and reversible contact with the underlying human skin surface. A mechanics model is developed in this note to extend the experimental design of Dagdeviren et al. [16] for simultaneous determination of the Young's moduli of the epidermis and dermis when the system is not perfectly bonded to the human skin.

Model Description and Analytical Solutions. As shown in Fig. 1, the human skin is modeled as a two-layered structure composed of an epidermis layer and a dermis layer. Each layer is assumed to be linear elastic and isotropic. The dermis layer (thickness ~ 1 mm [17]) is much thicker than the epidermis layer (thickness ~ 0.1 mm [17]) and is therefore modeled as a semi-infinite solid. The thickness of the epidermis layer is denoted as H. The Young's moduli of the epidermis and dermis layers are $E_{\mathrm{epidermis}}$ and E_{dermis}, respectively, and their Poisson's ratios are 0.5 because of their incompressibility.

The encapsulated piezoelectric sensors and actuators, if not perfectly bonded to the human skin, may slip along the skin surface, which may significantly reduce the interfacial shear stress. The deformation of the actuators/sensors is then dominated by the normal stress, not shear stress, at the interface. This leads to the change in the actuator/sensor thickness, which can be modeled by

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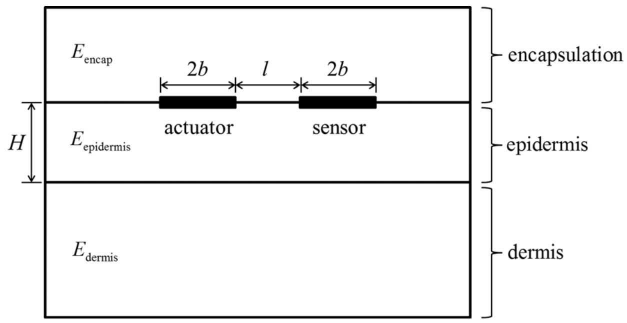

Fig. 1 Two-layered model of the human skin and skin-modulus device mounted on the surface of human skin with encapsulation

a pair of edge dislocations embedded in the surrounding media for each actuator/sensor.

The piezoelectric, dielectric, and elastic constants of the actuators and sensors normal to the skin are e_{33}, k_{33}, and c_{33}, respectively. The thick encapsulation layer is also modeled as a semi-infinite, linear-elastic solid with the Young's modulus E_{encap} and Poisson's ratio of 0.5 [10,16]. Figure 1 shows an actuator and a sensor (made of identical materials), with length $2b$ and spacing l. A two-dimensional model for plane-strain deformation is adopted for Fig. 1.

The actuator is subjected to an input voltage U_{input}. Without the surrounding media, its thickness would increase by

$$\Delta = A\frac{e_{33}}{c_{33}}U_{\mathrm{input}} \tag{1}$$

where $A = (c_{33}/e_{33})\cdot(c_{11}e_{33} - c_{13}e_{31})/(c_{11}c_{33} - c_{13}^2)$ is given in terms of the piezoelectric constants e_{ij} and elastic constants c_{ij}. The thickness increase induces deformation in the surrounding encapsulation, epidermis, and dermis, though their Young's moduli ($\sim$100 kPa [16]) are many orders of magnitude smaller than that of the actuator ($\sim$100 GPa [16]). Consequently, the decrease of the actuator thickness due to the constraint of the surrounding media is negligible as compared to Δ [10,16].

The thickness of the actuators and sensors ($\sim$5 μm [16]) is much smaller than that of the epidermis (and dermis and encapsulation). Therefore, Eq. (1) can be modeled as a pair of edge dislocations, with the Burgers vector Δ and $-\Delta$, separated by the length $2b$ of the actuator on the encapsulation/epidermis interface for a finite-thickness epidermis sandwiched by semi-infinite encapsulation and dermis. The deformation and stress in the encapsulation, epidermis, and dermis are obtained analytically, similar to the studies of dislocations in layered media [18,19].

Because the sensors are extremely thin and stiff, their thickness changes are essentially zero such that they do not need to be modeled as pairs of dislocations. The normal stress $\bar{\sigma}$ averaged over length $2b$ of the sensor (at the spacing l from the actuator) is obtained analytically by [18–20]

$$\bar{\sigma} = \frac{\Delta}{2b}\frac{E_{\mathrm{epidermis}}}{3\pi}(1-\alpha_{\mathrm{encap}}) \times\left[\ln\frac{(2b+l)^2}{(4b+l)l} + (1-\alpha_{\mathrm{encap}})\alpha_{\mathrm{dermis}}(f_1 + \alpha_{\mathrm{encap}}\alpha_{\mathrm{dermis}}f_2)\right] \tag{2}$$

where $\alpha_{\mathrm{encap}} = (E_{\mathrm{epidermis}} - E_{\mathrm{encap}})/(E_{\mathrm{epidermis}} + E_{\mathrm{encap}})$ and $\alpha_{\mathrm{dermis}} = (E_{\mathrm{epidermis}} - E_{\mathrm{dermis}})/(E_{\mathrm{epidermis}} + E_{\mathrm{dermis}})$ are the first Dundurs' parameter [21] for the encapsulation/epidermis and epidermis/dermis interfaces, respectively, and f_1 and f_2 are the nondimensional functions given by

$$f_1 \approx \frac{1}{2}\ln\frac{\theta_{0,1}\theta_{2,1}}{\theta_{1,1}^2} + \frac{1}{2}\left(2\theta_{1,1}^{-1} - \theta_{0,1}^{-1} - \theta_{2,1}^{-1}\right) + \left(2\theta_{1,1}^{-2} - \theta_{0,1}^{-2} - \theta_{2,1}^{-2}\right) \tag{3a}$$

and

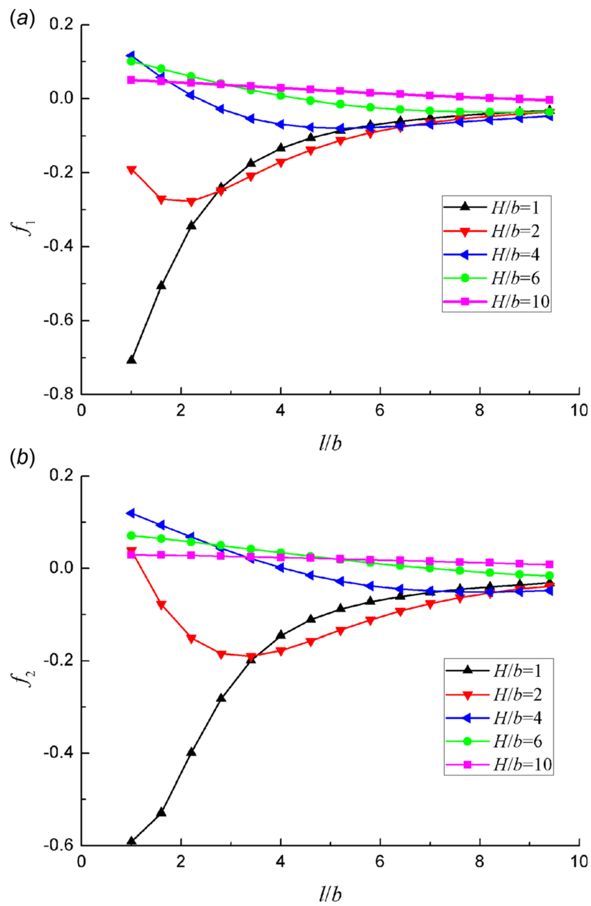

Fig. 2 (a) f_1 versus l/b and (b) f_2 versus l/b

$$f_2 \approx \frac{1}{2}\ln\frac{\theta_{0,2}\theta_{2,2}}{\theta_{1,2}^2} + \frac{1}{2}\left(2\theta_{1,2}^{-1} - \theta_{0,2}^{-1} - \theta_{2,2}^{-1}\right) + \frac{7}{16}\left(2\theta_{1,2}^{-2} - \theta_{0,2}^{-2} - \theta_{2,2}^{-2}\right) - \frac{1}{2}\left(2\theta_{1,2}^{-3} - \theta_{0,2}^{-3} - \theta_{2,2}^{-3}\right) + \frac{3}{2}\left(2\theta_{1,2}^{-4} - \theta_{0,2}^{-4} - \theta_{2,2}^{-4}\right) \tag{3b}$$

in which

$$\theta_{m,n} = 1 + \frac{(l+2mb)^2}{(2nH)^2} \tag{4}$$

f_1 and f_2 are shown versus l/b in Fig. 2 for different values of H/b; they approach zero for $H/b \gg 1$ (small sensor $2b$ as compared to the thickness H of epidermis).

The output voltage of the sensor is related to the normal stress $\bar{\sigma}$ in Eq. (2) by

$$U_{\mathrm{output}} = \frac{1}{S}\frac{h_{\mathrm{piezo}}}{e_{33}}\bar{\sigma} \tag{5}$$

where $S = 1 + (e_{31}/e_{33})\cdot(c_{33}e_{31} - c_{13}e_{33})/(c_{11}e_{33} - c_{13}e_{31}) + (k_{33}/e_{33})\cdot(c_{11}c_{33} - c_{13}^2)/(c_{11}e_{33} - c_{13}e_{31})$ is given in terms of the piezoelectric constants e_{ij}, elastic constants c_{ij}, and dielectric constants k_{ij}, and h_{piezo} is the thickness of the piezoelectric layer in the sensor.

The ratio of the sensor output voltage in Eq. (5) to the actuator input voltage in Eq. (1) gives

$$\frac{U_{\mathrm{output}}}{U_{\mathrm{input}}} = \frac{1}{3\pi}\frac{A}{S}\frac{h_{\mathrm{piezo}}}{2b}\frac{E_{\mathrm{epidermis}}}{c_{33}}(1-\alpha_{\mathrm{encap}}) \times\left[\ln\frac{(2b+l)^2}{(4b+l)l} + (1-\alpha_{\mathrm{encap}})\alpha_{\mathrm{dermis}}(f_1 + \alpha_{\mathrm{encap}}\alpha_{\mathrm{dermis}}f_2)\right] \tag{6}$$

This ratio for a system poorly bonded to the skin is expected to be much smaller than that for a perfectly bonded case. Equation (6) is explored in the following to develop a strategy for determining $E_{\text{epidermis}}$, E_{dermis}, and H in experiments.

Determination of the Young's Modulus of the Epidermis. For small actuators and sensors with length $2b_{\text{small}}$ and spacing l_{small} less than 1/5 of the thickness of epidermis, the functions f_1 and f_2 are approximately zero, i.e., the effect of the dermis is negligible. Equation (6) then gives the Young's modulus of the epidermis as

$$E_{\text{epidermis}} = \left[\frac{2}{3\pi}\frac{A}{S}\frac{h_{\text{piezo}}}{2b_{\text{small}}}\frac{1}{c_{33}}\left(\frac{U_{\text{output}}}{U_{\text{input}}}\right)^{-1}_{\text{small}} \times \ln\frac{(2b_{\text{small}}+l_{\text{small}})^2}{(4b_{\text{small}}+l_{\text{small}})l_{\text{small}}} - \frac{1}{E_{\text{encap}}}\right]^{-1} \quad (7)$$

For a known Young's modulus E_{encap} of the encapsulation, $E_{\text{epidermis}}$ can be determined from the experiment for small actuators and sensors, independent of E_{dermis} and H.

Determination of the Young's Modulus of the Dermis. For lengths or spacing of actuators and sensors that are not necessarily small as compared to the thickness of epidermis, the thickness of epidermis and the moduli of both the epidermis and dermis come into play. For an actuator and two sensors having the same length $2b$ but different actuator–sensor spacing l_1 and l_2, Eq. (6) becomes

$$\left(\frac{U_{\text{output}}}{U_{\text{input}}}\right)_i = \frac{1}{3\pi}\frac{A}{S}\frac{h_{\text{piezo}}}{2b}\frac{E_{\text{epidermis}}}{c_{33}}(1-\alpha_{\text{encap}}) \times \left\{\ln\frac{(2b+l_i)^2}{(4b+l_i)l_i} + (1-\alpha_{\text{encap}})\alpha_{\text{dermis}} \times \left[(f_1)_i + \alpha_{\text{encap}}\alpha_{\text{dermis}}(f_2)_i\right]\right\} \quad (8)$$

where $(U_{\text{output}}/U_{\text{input}})_i$ is obtained from the experiments for sensor i, $(f_1)_i$ and $(f_2)_i$ are f_1 and f_2 in Eq. (3) for spacing l_i, and $E_{\text{epidermis}}$ obtained from Eq. (7) also gives α_{encap}. Equation (8) constitutes two equations for α_{dermis} and H, which can be solved numerically, and α_{dermis} then gives E_{dermis}.

The Young's moduli of the epidermis and dermis can be obtained simultaneously by applying a system consisting of actuators and sensors with small lengths and spacing ($2b_{\text{small}}$, l_{small}) and ones ($2b$, l) comparable to the thickness of epidermis.

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