Axisymmetric thermo-mechanical analysis of laser-driven non-contact transfer printing

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Abstract An axisymmetric thermo-mechanical model is developed for laser-driven non-contact transfer printing, which involves laser-induced impulsive heating to initiate separation at the interface between a soft, elastomeric stamp and hard micro/nanomaterials (i.e., inks) on its surface, due to a large mismatch in coefficients of thermal expansion. The result is the active ejection of the inks from the stamp, to a spatially separated receiving substrate, thereby representing the printing step. The model gives analytically the temperature field, and also a scaling law for the energy release rate for delamination at the interface between the stamp and an ink in the form of a rigid plate. The normalized critical laser pulse time for interfacial delamination depends only on the normalized absorbed laser power and width of the ink structure, and has been verified by experiments.

Keywords Laser-driven non-contact transfer printing · Thermo-mechanical analysis · Axisymmetric model · Stamp and ink

1 Introduction

Laser-driven non-contact transfer printing is a new technique for materials assembly and micro-/nano-fabrication. Micro/nanoscale materials, such as wires, membranes, platelets, etc, are retrieved (i.e., picked up) from a growth (donor) substrate via an elastomeric
stamp, and then printed onto a different (receiver) substrate without direct contact (Meitl et al. 2006). A laser pulse initiates separation at the adhesive surface due to large thermal mismatch between the stamp and ink.

As illustrated in Fig. 1, the process starts with retrieval of inks from a donor substrate (Fig. 1a, b) with an elastomeric (e.g., PDMS). The ‘inked’ stamp is then brought close (a few micrometers) to the receiving substrate (Fig. 1c). A pulsed laser beam, focusing on the stamp/ink interface, causes the active ejection of the inks from the stamp such that they land on the receiving substrate (Fig. 1d).

Laser-driven non-contact transfer printing can be valuable for advanced engineering development of systems such as stretchable/flexible semiconductor devices for structural health monitoring (Nathan et al. 2000), image sensors (Lumelsky et al. 2001; Mannsfeld et al. 2010; Someya et al. 2005; Someya and Sekitani 2009), flexible display (Crawford 2005; Forrest 2004; Gelinck et al. 2004), deformable circuits (Kim et al. 2008; Sekitani et al. 2010) flexible inorganic solar cells (Yoon et al. 2008) and LEDs (Sekitani et al. 2009). It can integrate high-performance inorganic semiconductor materials, in ultrathin geometries, with substrates of interest, such as sheets of plastic or slabs or rubber. The resulting components can be of particular value in biomedical devices such as smart surgical gloves (Someya et al. 2004), in biomimetic, curvilinear electronics (Ko et al. 2008), bio-dissolvable electronics (Kim et al. 2010a), monitors for cardiac electrophysiology (Viventi et al. 2010) and ablation therapy (Kim et al. 2011a), foldable electrode arrays for mapping brain activity (Viventi et al. 2011), waterproof optoelectronics for diagnostics (Kim et al. 2010b), and epidermal electronics for health/wellness evaluation and brain-machine interfaces (Kim et al. 2011b).

A two-dimensional (plane-strain) thermo-mechanical model for laser-driven non-contact transfer printing (Li et al. 2012) has identified that the mismatch between the thermo-mechanical properties of the stamp and ink drives their interfacial delamination. The purpose of this paper is to develop a more realistic, axisymmetric thermo-mechanical model for laser-driven non-contact transfer printing. A scaling law is established to identify a single non-dimensional combination of material and geometry parameters that control laser-driven non-contact transfer printing. The scaling law agrees very well with the experiments, and is useful to optimal design of stamp for transfer printing.

2 Heat conduction

An axisymmetric model of heat conduction is established for the laser-driven non-contact transfer printing illustrated in Fig. 1c. The ink has radius $r_{\text{ink}}$ and thickness $h_{\text{ink}}$ attached to a large elastomeric stamp, which is modeled as a semi-infinite solid (Saeidpourazar et al. 2012). The absorbed laser power by the ink is denoted by $P$.

The transient heat conduction equation in the stamp is (Incropera et al. 2007)

$$\frac{\partial^2 \theta_{\text{stamp}}}{\partial r^2} + \frac{1}{r} \frac{\partial \theta_{\text{stamp}}}{\partial r} + \frac{\partial^2 \theta_{\text{stamp}}}{\partial z^2} - \frac{c_{\text{stamp}} \rho_{\text{stamp}}}{\lambda_{\text{stamp}}} \frac{\partial \theta_{\text{stamp}}}{\partial t} = 0$$

with the initial condition $\theta_{\text{stamp}}|_{t=0} = 0$, where $(r, z)$ are the cylindrical coordinates (Fig. 1b), $t$ is time, $\theta_{\text{stamp}}$ is the temperature increase from the ambient temperature, and $c_{\text{stamp}}$, $\rho_{\text{stamp}}$ and $\lambda_{\text{stamp}}$ are the specific heat, mass density and thermal conductivity of the stamp, respectively. The natural convection on the stamp surface is negligible because of the short duration of pulsed laser beam. This gives the boundary condition

\[ \text{Boundary Condition} \]
\[-\lambda_{\text{stamp}} \frac{\partial \theta_{\text{stamp}}}{\partial z} \bigg|_{z=0} = \begin{cases} q(t) & \text{for } 0 \leq r \leq r_{\text{ink}} \\ 0 & \text{for } r > r_{\text{ink}} \end{cases}, \tag{2}\]

where \( q(t) \) is the heat flux into the stamp. It is obtained approximately in the “Appendix” as

\[ q(t) = \frac{P}{\pi r_{\text{ink}}^2} \left[ 1 - \exp \left( \frac{t}{t_0} \right) \right. \left. \text{erfc} \left( \sqrt{\frac{t}{t_0}} \right) \right], \tag{3} \]

where \( \text{erfc} \) is the complementary error function (Fettis et al. 1973),

\[ t_0 = \frac{c_{\text{ink}}^2 \rho_{\text{ink}} h_{\text{ink}}^2}{c_{\text{stamp}} \rho_{\text{stamp}} \lambda_{\text{stamp}}} \tag{4} \]

represents the characteristic time in laser-driven non-contact transfer printing, and \( c_{\text{ink}} \) and \( \rho_{\text{ink}} \) are the specific heat and mass density of the ink, respectively.

The Hankel transform and Fourier cosine transform (with respect to \( r \) and \( z \), respectively) of Eq. (1) gives an ordinary differential equation with respect to \( t \). Its solution is obtained analytically. The inverse transforms give the temperature increase in the stamp as

\[
\begin{align*}
\theta_{\text{stamp}}(r, z, t) &= \frac{c_{\text{ink}} \rho_{\text{ink}} h_{\text{ink}}}{c_{\text{stamp}} \rho_{\text{stamp}} \lambda_{\text{stamp}}} \frac{P}{\sqrt{\pi} r_{\text{ink}}^2} \\
&\times \left[ \int_0^{t'} \frac{1 - \exp \left( \frac{(t' - \tau)}{\sqrt{t'}} \right) \text{erfc} \left( \frac{-
\frac{\tau^2}{4t'}}{r_{\text{ink}}^2} \right)}{\sqrt{\tau}} \, d\tau \right. \\
&\left. \times \int_0^\infty J_0 (\eta) J_1 (\eta) \exp \left( -\frac{\eta^2}{r_{\text{ink}}^2} \right) \, d\eta \right], \tag{5}
\end{align*}
\]

where \( \eta = r_{\text{ink}} u \) and \( \tau = \frac{z}{r_{\text{ink}}} \) are the normalized coordinates, \( t' = t / t_0 \) is the normalized time, and

\[ r_{\text{ink}}' = \frac{c_{\text{stamp}} \rho_{\text{stamp}} r_{\text{ink}}}{c_{\text{ink}} \rho_{\text{ink}} h_{\text{ink}}} \tag{6} \]

is the normalized radius of the ink.

Equation (5) suggests that the temperature increase, normalized by \( c_{\text{ink}} \rho_{\text{ink}} h_{\text{ink}} P / (c_{\text{stamp}} \rho_{\text{stamp}} \lambda_{\text{stamp}} r_{\text{ink}}^2) \), depends on the normalized position \( (r' \) and \( \tau' \) and time \( (t') \), and a single combination of stamp and ink properties \( r_{\text{ink}}' = c_{\text{stamp}} \rho_{\text{stamp}} r_{\text{ink}} / (c_{\text{ink}} \rho_{\text{ink}} h_{\text{ink}}) \). Figure 2 shows the normalized distribution of temperature increase in the radial direction for \( \eta = \tau / r_{\text{ink}} = 0, 0.1 \) and \( 0.2 \), normalized time \( t' = 8.90 \) and \( r_{\text{ink}} = 16.4 \), which corresponds to \( c_{\text{ink}} = 708 \, \text{J kg}^{-1} \text{K}^{-1}, \ 
\rho_{\text{ink}} = 2.30 \times 10^3 \, \text{kg m}^{-3} \) for silicon (Campbell 2001), \( c_{\text{stamp}} = 1.46 \, \text{kJ kg}^{-1} \text{K}^{-1}, \ 
\rho_{\text{stamp}} = 970 \, \text{kg m}^{-3} \) for PDMS (Mark 1999), and \( r_{\text{ink}} = 56.4 \mu \text{m} \) and \( h_{\text{ink}} = 3 \mu \text{m} \) in the experiment (Saeidpourazar et al. 2012).

Finite element method (FEM) is used to obtain the temperature distributions in the stamp and ink. The ink is modeled as a heat source with the total heat flux \( \frac{P}{\pi r_{\text{ink}}^2} \). As shown in Fig. 2, the numerical results agree reasonably with the analytical expression in Eq. (5). A contour plot of the temperature field in the cross section cut along the symmetry axis \( z \) is included as an inset in Fig. 2.

The temperature increase in the ink is nearly uniform because its thermal conductivity \( \lambda_{\text{ink}} \) (e.g., 160 W m\(^{-1}\)K\(^{-1}\) for silicon) (Campbell 2001) is three orders of magnitude larger than \( \lambda_{\text{stamp}} \) (e.g., 0.15 W m\(^{-1}\)K\(^{-1}\) for PDMS) (Mark 1999). It is obtained analytically as

\[
\theta_{\text{ink}} (t) = \frac{c_{\text{ink}} \rho_{\text{ink}} h_{\text{ink}}}{c_{\text{stamp}} \rho_{\text{stamp}} \lambda_{\text{stamp}}} \\
\times \frac{2P}{\sqrt{\pi} r_{\text{ink}}^2} \int_{0}^{t'} \frac{1 - \exp \left( \frac{(t' - \tau)}{\sqrt{\tau}} \right) \text{erfc} \left( \frac{-
\frac{\tau^2}{4t'}}{r_{\text{ink}}^2} \right)}{\sqrt{\tau}} \, d\tau \\
\times \int_{0}^{\infty} J_1 (\eta) \exp \left( -\frac{\eta^2}{r_{\text{ink}}^2} \right) \, d\eta. \tag{7}
\]

The normalized temperature increase in the ink, \( \frac{c_{\text{stamp}} \rho_{\text{stamp}} \lambda_{\text{stamp}} r_{\text{ink}}^2 \theta_{\text{ink}}}{c_{\text{ink}} \rho_{\text{ink}} h_{\text{ink}} P} \), versus the normalized time \( t' \) depends only on the same single combination of stamp and ink properties as in Eq. (6).

As shown in Fig. 3 for the same properties and parameters as in Fig. 2, Eq. (7) agrees well with the numerical results obtained by FEM.
3 Stamp/ink interfacial delamination

The temperature increase in Sect. 2 yields the thermal strain $\alpha_{\text{stamp}}\theta_{\text{stamp}}$ in the stamp, which drives delamination of the stamp/ink interface, where $\alpha_{\text{stamp}}$ is the coefficient of thermal expansion (CTE) of the stamp (e.g., $3.1 \times 10^{-4}\text{K}^{-1}$ for PDMS) (Mark 1999). The CTE of the ink (e.g., $2.6 \times 10^{-6}\text{K}^{-1}$ for silicon) (Okada and Tokumaru 1984) is two orders of magnitude smaller such that the thermal strain in the ink is negligible for stamp/ink interfacial delamination.

For a plane-strain, interfacial crack that lies on $x < 0$, $z = 0$ between two dissimilar materials subjected to a transformation strain over an infinitesimal area $dx dz$ around a point $(x, z)$ ($z > 0$), the complex stress intensity factor is $\sqrt{2/\pi}\mu_{\text{stamp}}(\varepsilon_{xx}^T + \varepsilon_{zz}^T)(x - iz)^{-3/2} dx dz$ (Suo 1989), where $i = \sqrt{-1}$, $\varepsilon_{xx}$ and $\varepsilon_{zz}$ are the transformation strains, $\mu_{\text{stamp}}$ is the shear modulus of the stamp, which is five orders of magnitude smaller than that of the ink such that the latter can be considered as rigid (Lu et al. 2007). For an axisymmetric interfacial crack between the stamp and the rigid ink subject to a thermal strain $\alpha_{\text{stamp}}\theta_{\text{stamp}}$ over a ring with the infinitesimal cross section area $dr dz$ around $(r, z)$ ($z > 0$), the complex stress intensity factor takes the form $\mu_{\text{stamp}}\alpha_{\text{stamp}}\theta_{\text{stamp}}\rho_{\text{ink}}^{-5/2} f \left(\frac{r}{r_{\text{ink}}}, \frac{z}{r_{\text{ink}}}\right) r dr dz$, where $f$ is a non-dimensional function. Its integration over the stamp gives the complex stress intensity factor as

$$K = \int_0^\infty \int_0^\infty \theta_{\text{stamp}} (r, z, t) \cdot r dr dz,$$

Substitution of Eq. (5) into the above expression gives

$$K = \sqrt{\frac{c_{\text{stamp}}\rho_{\text{stamp}}}{c_{\text{ink}}\rho_{\text{ink}}}} \mu_{\text{stamp}}\alpha_{\text{stamp}} P \lambda_{\text{stamp}} \int_0^t \frac{t - t'}{r_{\text{ink}}^3} \cdot \mathcal{K} \left(\frac{t - t'}{r_{\text{ink}}}\right),$$

where $\mathcal{K}$ is a non-dimensional function of only two variables, the normalized time and ink radius. The interfacial crack tip energy release rate (Suo 1989) $G = |K|^2/(8\mu_{\text{PDMS}})$ is given by

$$G = \frac{c_{\text{stamp}}\rho_{\text{stamp}}\mu_{\text{stamp}}\alpha_{\text{stamp}}^2 P^2}{8c_{\text{ink}}\rho_{\text{ink}} h_{\text{ink}} \lambda_{\text{stamp}}^2} \cdot \mathcal{G} \left(\frac{t - t'}{r_{\text{ink}}}\right),$$

where $\mathcal{G} \left(\frac{t - t'}{r_{\text{ink}}}\right) = \left|\mathcal{K} \left(\frac{t - t'}{r_{\text{ink}}}\right)\right|^2$ is also a non-dimensional function of the normalized time and ink radius.

4 Scaling law for the laser pulse time for delamination

The stamp/ink interface delaminates when the interfacial crack tip energy release rate in Eq. (10) reaches the work of adhesion $\gamma$ of the interface, $G = \gamma$, which, together with Eq. (10), give the critical time $t_{\text{delamination}}$ for stamp/ink interfacial delamination,

$$t_{\text{delamination}} = \tilde{t} \left(\frac{P}{P_0}, \frac{r_{\text{ink}}}{r_0}\right),$$

where $\tilde{t}$ is a non-dimensional function, $t_0$ is given in Eq. (4), and

$$P_0 = \frac{\lambda_{\text{PDMS}}}{\alpha_{\text{PDMS}}} \left[\gamma c_{\text{silicon}} \rho_{\text{silicon}} h_{\text{silicon}} \right] \frac{1}{c_{\text{PDMS}}/\rho_{\text{PDMS}} \mu_{\text{PDMS}}},$$

$$r_0 = \frac{c_{\text{silicon}} \rho_{\text{silicon}} h_{\text{silicon}}}{c_{\text{PDMS}}/\rho_{\text{PDMS}}}.$$
The element CAX8RT for coupled thermal-mechanical analysis in the ABAQUS finite element program (Dassault Systèmes 2009) is used. The crack is located at the end of the stamp/ink interface with the initial crack length 0.1 μm. The boundary condition for the top surface of the stamp is free convection with the coefficient of natural convection 25 W m$^{-2}$ K$^{-1}$ (Incropera et al. 2007), while other surfaces are thermal insulated. For a specified work of adhesion, the critical time for different total heat flux is obtained. The critical time is shown versus the laser power in Fig. 4 according to the scaling law in Eq. (11). The laser pulse time of 1, 2, 3 and 4 ms in the experiment correspond to absorbed laser power by the ink (100 × 100 × 3 μm polished silicon chip) 0.0672, 0.0403, 0.0269 and 0.0222 W, respectively, which give the total heat flux 6.72 × 10$^6$, 4.03 × 10$^6$, 2.69 × 10$^6$ and 2.22 × 10$^6$ W m$^{-2}$. The scaling law agrees very well with the experiments (Saeidpourazar et al. 2012) also shown in Fig. 4.

5 Concluding remarks

Laser-driven non-contact transfer printing involves many thermal and mechanical properties of the stamp, ink, their interface, and the thickness and radius of the ink, and the absorbed laser power $P$ by the ink. A scaling law is established for the critical laser pulse time $t_{delamination}$ for stamp/ink interfacial delamination; the normalized $t_{delamination}$ depends only on the normalized power $P$ and radius of the ink $r_{ink}$. All materials properties and the thickness of the ink $h_{ink}$ come into play only via the normalizations of $t_{delamination}$, $P$ and $r_{ink}$. 

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Appendix: Heat flux into the stamp

An approximate heat conduction model $\frac{\partial^2 \theta_{\text{stamp}}}{\partial \tau^2} - \frac{c_{\text{stamp}} \rho_{\text{stamp}}}{\lambda_{\text{stamp}}} \frac{\partial \theta_{\text{stamp}}}{\partial \tau} = 0$ is adopted to estimate the heat flux into the stamp $q(t)$. The Fourier cosine transform $\tilde{\theta}_{\text{stamp}} = \int_0^\infty \theta_{\text{stamp}}(z) \cos(\beta z) \, dz$, together with the boundary condition in Eq. (2), give

$$q(t) = \frac{\lambda_{\text{stamp}} \beta^2}{c_{\text{stamp}} \rho_{\text{stamp}}} \left[ \int_0^\infty \frac{\lambda_{\text{stamp}} \beta^2}{c_{\text{stamp}} \rho_{\text{stamp}}} (t - \tau) \exp \left( -\frac{\lambda_{\text{stamp}} \beta^2}{c_{\text{stamp}} \rho_{\text{stamp}}} (t - \tau) \right) \right] \, d\tau. \quad (A1)$$

The inverse Fourier cosine transform then gives

$$\theta_{\text{stamp}}(z=0) = \frac{1}{\sqrt{\pi c_{\text{stamp}} \rho_{\text{stamp}} \lambda_{\text{stamp}}}} \int_0^t \frac{q(\tau)}{\sqrt{t-\tau}} \, d\tau. \quad (A2)$$

The temperature increase (from the ambient temperature) in the ink $\theta_{\text{ink}}$ is nearly uniform because its thermal conductivity $\lambda_{\text{ink}}$ (e.g., 160 W m$^{-1}$ K$^{-1}$ for silicon) (Campbell 2001) is three orders of magnitude larger than $\lambda_{\text{stamp}}$ (0.15 W m$^{-1}$ K$^{-1}$) (Mark 1999). Its rate of increase is related to the heat flux into the ink $P/(\pi r_{\text{ink}}^2)$ by

$$\frac{d\theta_{\text{ink}}}{dt} = \frac{P/(\pi r_{\text{ink}}^2) - q(t)}{c_{\text{ink}} h_{\text{ink}} r_{\text{ink}}}. \quad (A4)$$

Continuity of temperature across the stamp/ink interface requires

$$\theta_{\text{ink}} = \theta_{\text{stamp}}|_{z=0} \quad \text{for} \quad 0 \leq r \leq r_{\text{ink}}. \quad (A5)$$
Equations (A3)–(A5) give

\[ \frac{1}{\sqrt{\pi} c_{\text{stamp}}} \rho_{\text{stamp}} \lambda_{\text{stamp}} \int_0^t \frac{q(\tau)}{\sqrt{t - \tau}} d\tau = \frac{1}{c_{\text{ink}}} \rho_{\text{ink}} h_{\text{ink}} \left[ \frac{P}{\pi r_{\text{ink}}^2} - \int_0^t \frac{q(\tau)}{\sqrt{t - \tau}} d\tau \right]. \tag{A6} \]

which is solved by the Laplace transform to give the solution in Eq. (3).

References


