Local versus global buckling of thin films on elastomeric substrates

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Local buckling can form microcorrugations in thin films on elastomeric substrates, to yield an effective type of mechanical stretchability in otherwise rigid, brittle materials, with many application possibilities. For large area films or relatively thin substrates, however, global (Euler) buckling, as opposed to local buckling, can be observed in experiments. This paper describes analytically the mechanics of local and global buckling of one-dimensional thin films or two-dimensional thin membranes on elastomeric substrates. The critical condition separating these two buckling modes is obtained analytically, and it agrees well with experiments and numerical simulations. © 2008 American Institute of Physics. [DOI: 10.1063/1.2956402]

Thin films on elastomeric substrates are important for electronics systems that require or benefit from mechanical stretchability, such as flexible displays,1 electronic eye camera,2 conformable skin sensors,3 smart surgical gloves,4 and structural health monitoring devices.5 Other emerging applications include micro- and nanoelectromechanical systems,6 tunable phase optics,7,8 force spectroscopy in cells,9 biocompatible topographic matrices for cell camera,2 conformable skin sensors,3 smart surgical gloves,4 and stretchability, such as flexible displays,1 electronic eye composite beam is [EA=\frac{E_s h_s}{h_f}+\frac{E_f h_f}{h_f}], while the effective bending rigidity is \[
\frac{1}{EI} = \frac{(E_s h_s^2 - E_f h_f^2)^2 + 4E_s h_s h_f (h_s + h_f)^2}{12(E_s h_s + E_f h_f)}.
\]

Another buckling mode, namely, the global buckling shown in Fig. 1(c), has been observed in experiments for cases where the substrate is relatively thin. Although this behavior can be eliminated by adding thin layers of substrate material on top of the system,29 an understanding of the physics is important for this emerging field of technology. The objective of this letter is to identify the critical condition separating the local and global buckling modes for an elastic film on an elastomeric substrate. The critical strain for global buckling \(e_{\text{critical}}^{\text{local}}\), to be given in the following, is compared to the local buckling strain \(e_{\text{critical}}^{\text{local}}\) in Eq. (1) to determine the critical condition separating the two buckling modes.

The film and substrate are modeled as a composite beam in global buckling. The effective tensile rigidity of the composite beam is \(\frac{E_s h_s}{h_f}+\frac{E_f h_f}{h_f}\), while the effective bending rigidity is \(\frac{1}{EI} = \frac{(E_s h_s^2 - E_f h_f^2)^2 + 4E_s h_s h_f (h_s + h_f)^2}{12(E_s h_s + E_f h_f)}\). The

![Schematic illustration of the process for fabricating buckled single crystal Si ribbons (yellow) on a PDMS (gray) substrate: (a) bond Si elements to prestretched PDMS; (b) local buckling; and (c) global buckling due to release of prestretch.](image)

\[A_0 = h_f \sqrt{\frac{E_s h_f}{E_f}} - 1\]

of the local buckling profile (inset of Fig. 2), which are both proportional to the film thickness \(h_f\), the amplitude increases with the prestrain, and

\[e_{\text{critical}}^{\text{local}} = \frac{1}{4} \left(\frac{3E_s}{E_f}\right)^{2/3}\]

is the critical buckling strain given in terms of the plane-strain elastic moduli \(E_s\) and \(E_f\) of the film and substrate. The wavelength and amplitude agree well with experiments,23 and have been used as in modern metrology.12-15 The substrate thickness \(h_s\) does not come into play because the substrate is much thicker than the film and therefore is considered a semi-infinite solid.

\[E_s = 1.0 \times 10^6 \text{MPa}, \ E_f = 2.0 \times 10^{-5} \text{MPa}\]

\[h_s = 0.001 \text{m}, \ h_f = 0.0001 \text{m}\]

\[A_0 = 1.0\]
critical buckling strain for a beam of length $L$ with clamped ends, accounting for the shear effect, is

$$\varepsilon_{\text{critical}} = \frac{1}{1 + \frac{1.2F_0^0}{\sqrt{EA}}}$$

for a rectangular cross section, where $F_0^0 = 4\pi^2 EI/L^2$ is the critical buckling load neglecting the effect of shear, and $G$ is the effective shear modulus of the composite beam, which is approximately the shear modulus $G_s$ of the substrate since the film is very stiff.

Local buckling occurs when its critical strain in Eq. (1) is smaller than its counterpart in Eq. (2) for global buckling, i.e., $\varepsilon_{\text{critical}}^{\text{local}} < \varepsilon_{\text{critical}}^{\text{global}}$ while global buckling occurs when the opposite holds, $\varepsilon_{\text{critical}}^{\text{local}} > \varepsilon_{\text{critical}}^{\text{global}}$. This is equivalent to comparing the energy in local and global buckling. Figure 2 shows the critical local and global buckling strains versus the substrate thickness $h_s$ for a 1-μm-thick, 3-mm-long Si film (Young’s modulus $E_s = 130$ GPa, Poisson’s ratio $\nu_s = 0.27$) (Ref. 32) on a PDMS substrate ($E_f = 1.8$ MPa, $\nu_f = 0.5$). The global buckling strain (red curve) increases with the substrate thickness, while the local buckling strain remains a constant, $-0.034\%$ (blue line). For small substrate thickness such that $\varepsilon_{\text{critical}}^{\text{local}} > \varepsilon_{\text{critical}}^{\text{global}}$, global buckling occurs, while local buckling occurs for a thick substrate. The condition $\varepsilon_{\text{critical}}^{\text{local}} = \varepsilon_{\text{critical}}^{\text{global}}$, which corresponds to the intercept of two curves in Fig. 2, gives a critical length $L_{cr}$ separating local and global bucking as

$$L_{cr} = 4\pi \sqrt{\frac{EI}{E_s h_s + E_f h_f}} - \frac{0.3}{G_s (h_f + h_s)}$$

Local and global buckling occur for $L < L_{cr}$ and $L > L_{cr}$, respectively.

The finite element method is used to verify the above criterion for local and global buckings. Beam elements are used for the film, while the substrate is modeled by solid elements. The numerical results are shown in Fig. 2 (blue dots for local buckling and red dots for global buckling), and they agree very well with the analytical expressions (1) and (2).

Figure 3 shows the critical length normalized by film thickness, $L_{cr}/h_f$, versus the ratio of substrate to film thickness, $h_s/h_f$, for the Si film on PDMS substrate. This curve separates local buckling (below the curve) from global buckling (above the curve). Experimental results (solid circles) are also shown in Fig. 3 for film thickness $h_f = 300$ nm,
length $L = 15$ mm, and several substrate thicknesses $h_s = 0.4$, 0.6, 1.0, 2.0, and 4.0 mm. Local buckling is observed for all substrate thickness except for $h_s = 0.4$ mm, for which global buckling occurs. For $h_s = 300$ nm and $L = 15$ mm as in experiments, Fig. 3 gives a critical substrate thickness $h_s = 0.51$ mm separating the local and global buckling. This agrees very well with the experiments, $h_s = 0.4$ mm for global buckling and 0.6, 1.0, 2.0, and 4.0 mm for local buckling.

Local and global bucklings are also observed in two-dimensional Si thin membrane on PDMS substrate subject to equibiaxial compression. Thin silicon crystal membranes were prepared from silicon on insulator wafer [Soitec, Unibond; Si (300 nm)/SiO$_2$ (1000 nm)] and covalently bonded to various thickness PDMS, where the bonding was promoted by the UV/ozone treatment.33 The optical images in Fig. 4(b) show local buckling for membrane thickness $h_s = 1.0$, 2.0, and 4.0 mm, and global buckling for $h_s = 0.3$ mm. The Si membrane has length and width $L_1 \times L_2 = 10.4 \times 10.4$ mm$^2$, and thickness $h_s = 300$ nm. The critical strain for local buckling is $\varepsilon_{\text{local}}^{\text{critical}} = (3E_s/E_l)^{1/2}/(4(1 + \nu))$.34 For global buckling, the film and substrate are modeled as a composite plate with dimensions $L_1 \times L_2$, effective tensile rigidity $E_A$, and bending rigidity $E_l$. These give the critical strain for global buckling $\varepsilon_{\text{global}}^{\text{critical}} = \frac{4(1 - \nu)\pi E_A}{E_l L_1^2} \left( \frac{1}{E_s} + \frac{1}{E_l} \right)$.35 The critical length $L_{cr} = (L_1 = L_2)$ separating local and global buckling is determined from $\varepsilon_{\text{local}}^{\text{critical}} = \varepsilon_{\text{global}}^{\text{critical}}$, and is shown versus the ratio of substrate to membrane thickness, $h_s/h_s$, in Fig. 4(a). For $h_s = 300$ nm and $L_1 = L_2 = 10.4$ mm as in experiments, Fig. 4(a) gives a critical substrate thickness $h_s = 0.35$ mm below which global buckling occurs. This agrees very well with the experimental images in Fig. 4(b), which show global buckling for $h_s = 0.3$ mm and local buckling $h_s = 1.0$, 2.0, and 4.0 mm [also shown by solid triangles in Fig. 4(a)].

In summary, the critical condition for local and global buckling of thin films on compliant substrates is obtained analytically, and it agrees well with experimental and numerical results.

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